

# Fuzzy Hydrocyclone Modelling for Particle Separation Using Fuzzy Rule Interpolation

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**Abstract:** This paper reports on the use of a fuzzy rule interpolation technique for the modelling of hydrocyclones. Hydrocyclones are important equipment used for particle separation in mineral processing industry. Fuzzy rule based systems are useful in this application domains where direct control of the hydrocyclone parameters is desired. It has been reported that a rule extracting technique has been used to extract fuzzy rules from the input-output data. However, it is not uncommon that the available input-output dataset does not cover the universe of discourse. This results in the generation of sparse fuzzy rule bases. This paper examines the use of an improved multidimensional fuzzy rule interpolation technique to enhance the prediction ability of the sparse fuzzy hydrocyclone model. Fuzzy rule interpolation is normally used to provide interpretations from observations for which there are no overlaps with the supports of existing rules in the rule base.

## 1. Introduction

Mining and mineral processing are two important industries in Australia. The quality of the products depends heavily on the precise and efficient refinement and separation of the particles according to size and type. One of the most commonly used instruments for this purpose is the Hydrocyclone [1]. Hydrocyclones are used to classify and separate solids suspended in fluids, commonly known as *slurry*. The particles will leave the hydrocyclone through an underflow opening known as the *spigot*. On the other hand, an upward helical flow containing fine and lighter solid particles will exit via the vortex finder on top known as *upperflow*. For a hydrocyclone of fixed geometry, the performance of the system depends on a number of parameters. The separation efficiency of particles of a particular size is determined by an operational parameter known as *d50c*. This value indicates that 50% of particles of a particular size is reported to the upper and underflow streams.

The correct estimation of *d50c* is important since it is directly related to the efficiency of operations and it will also enable control of the hydrocyclone as illustrated by Gupta and Eren [2]. Computer control of hydrocyclones can be achieved by manipulation of operational parameters such as: diameter of the spigot opening ( $D_u$ ), the vortex finder height ( $H$ ), the inlet flowrate ( $Q_i$ ), the density ( $P_i$ ) and the

temperature ( $T$ ) of slurries for a desired  $d50c$ . Traditionally, mathematical models based on empirical methods and statistical techniques in describing the performance of the hydrocyclones are used. Although these approaches have long been established in the industry, they have their shortcomings. For example, the experimental conditions may vary, resulting in these empirical models being unreliable. Hence, the conventional approach may not be universally applicable. In recent years, Artificial Neural Network (ANN) [3, 4] and Neural-Fuzzy [5] techniques have been applied. Although ANN techniques have proven to be useful for the prediction of the  $d50c$  control parameter, the main disadvantage is their inability to convey the acquired knowledge to the user. As a trained network is represented by a collection of weights, the user will have difficulty in understanding and modifying the model. In many cases, the system may not gain the confidence of the user. The Neural-Fuzzy approach can be shown to be better than the ANN approach as it can generate fuzzy rules for the user to manipulate. However, the fuzzy rules generated to cover the whole sample space are too tedious for the user to examine.

In this paper, a fuzzy hydrocyclone model is proposed. By modifying the on-line control system shown in [2], the proposed fuzzy hydrocyclone model is shown in Figure 1. As in [2], the  $d50c$  is set to a desire value. The signals from the instruments are processed to calculate the present value of  $d50c$  using the conventional models. To minimise the differences between the set value and the present value, the operating parameters such as diameter of the spigot opening ( $D_u$ ), the vortex finder height ( $H$ ), and the inlet flowrate ( $Q_i$ ) are changed sequentially until the desired value of  $d50c$  is obtained. This is significant as the proposed technique allows users to manipulate the fuzzy rules easily, which also allows the system to perform in situations where no rules are found.

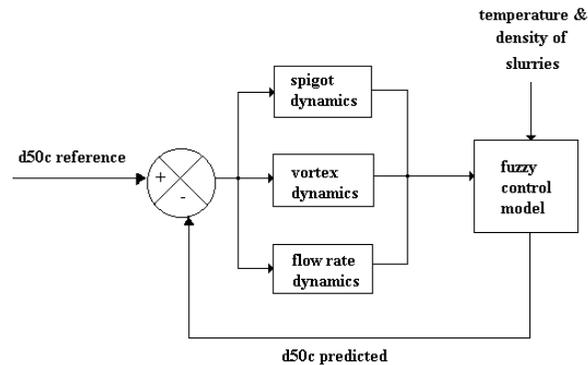


Figure 1: Online Fuzzy Hydrocyclone Control System

## 2. Fuzzy Hydrocyclone Control Model

Fuzzy control systems have shown to be useful in dealing with many control problems [6]. By far, they are the most important application of the classical fuzzy set theory. However, conventional fuzzy systems do not have any learning algorithms to build the analysis model. Rather, they are based on human or heuristic knowledge, past experience or detailed analysis of the available data in order to build the fuzzy rule base for the control system. Therefore, the major limitation is this difficulty in

building the fuzzy rules. Recently, an automatic self-generating fuzzy rules inference system [7] has shown successful results in establishing the well log interpretation model. This method is used in this paper to extract fuzzy rules from the test examples generated by the hydrocyclone model.

The steps involved in the self-generating fuzzy rules inference system are summarised as follows:

- (1) Determine the universe of discourse for each variable depending on its range of value.
- (2) Define the number of fuzzy regions and fuzzy terms for all data. For ease of extraction, only triangular types of membership functions are used.
- (3) The space associated with each fuzzy term over the universe of discourse for each variable is then calculated and divided evenly.
- (4) For each available test case, a fuzzy rule is established by directly mapping the physical value of the variable to the corresponding fuzzy membership function.
- (5) Go through Step (4) with all the available test cases and generate one rule for each input-output data pair.
- (6) Eliminate repeated fuzzy rules.
- (7) The set of remaining fuzzy rules together with the centroid defuzzification algorithm now forms the fuzzy interpretation model.

### 3. Problems of a Sparse Rule Base

To illustrate the problem of sparse rule base as described in the previous section, a practice case study is presented. Data collected from a Krebs hydrocyclone model D6B-120-839 have been used. There are a total of 70 training data and 69 testing data used in this study. The input parameters are  $Q_i$ ,  $P_i$ ,  $H$ ,  $D_u$ , and  $T$  and the output is  $d_{50c}$ . The self-generating fuzzy rules technique is used to extract fuzzy rules from the 70 training data. 7-membership function has been selected as it gives the best result. There are a total of 64 fuzzy sparse rules generated from the rule extraction process.

When this set of sparse rules are used to perform control on the testing data, 4 sets of data cannot find any fuzzy rules to fire and are shown in Figure 2. The output plot of the predicted  $d_{50c}$  (solid line on the plot) as compared to the observed  $d_{50c}$  (dots on the plot) is shown in Figure 3. The four zero output is the case where no rule fires. In this case study, the number of input sets that cannot find any rule to fire is considered minimal. However, in some cases, this may not always be true. If more than half the input instances cannot find any rule to fire, this control system may be considered useless. This is the major drawback for the fuzzy hydrocyclone control model. The problem also exists in most practical cases.

Warning: no rule is fired for input [493.0 26.00 85.20 3.750 26.00 ]! 0 is used as default output.  
 Warning: no rule is fired for input [388.0 24.20 69.50 2.650 33.00 ]! 0 is used as default output.  
 Warning: no rule is fired for input [462.0 10.20 85.20 3.750 40.00 ]! 0 is used as default output.  
 Warning: no rule is fired for input [267.00 24.50 85.20 3.750 34.00 ]! 0 is used as default output.

Figure 2: Warning message for input without firing rules.

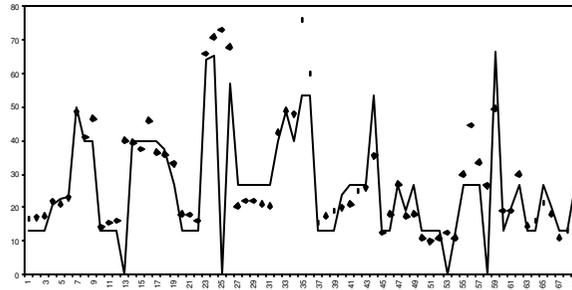


Figure 3: Output plot showing test case results indicating the rules fired.

#### 4. Fuzzy Rule Interpolation

In the case when a rule base contains gaps or is a sparse rule base, classical fuzzy reasoning methods can no longer be used. This is the problem highlighted in the previous section, as an observation finds no rule to fire. Fuzzy rule interpolation techniques provide a tool for specifying an output fuzzy set whenever at least one of the input universes is sparse. Kóczy and Hirota [8] introduced the first interpolation approach known as (linear) KH interpolation.

Two conditions can be applied for the use of linear interpolation. Firstly, there should exist an ordering on the input and output universes. This allows us to introduce a notion of distance between the fuzzy sets. Secondly, the input sets (antecedents, consequents and the observation) should be convex and normal fuzzy (CNF) sets. The method determines the conclusion by its  $\alpha$ -cuts in such a way that the ratio of distances between the conclusion and the consequents should be identical with the ones among the observation and the antecedents for all important  $\alpha$ -cuts (breakpoint levels).

The KH interpolation possesses several advantageous properties. Firstly, it behaves approximately linearly in between the breakpoint levels. Secondly, its computational complexity is low, as it is sufficient to calculate the conclusion for the breakpoint level set. However, for some input situations, it fails to result in a directly interpretable fuzzy set, because the slopes of the conclusion can collapse [9]. To address this problem, improved fuzzy rule interpolation techniques [9,10] have been developed. While most fuzzy interpolation techniques perform analysis on one-dimensional input space, the improved multidimensional fuzzy interpolation technique [11] proposed in this paper will handle multidimensional input spaces. This has been applied for the development of the fuzzy hydrocyclone control model.

#### 5. Case Study and Discussions

The test case described in this paper incorporates the improved multidimensional fuzzy interpolation method for the development of an accurate hydrocyclone model. As mentioned before, there are a total of four input instances that cannot find any firing rules (refer to Figure 2). From the observation and Euclidean distance measured on each input variable, the nearest fuzzy rules of the four input instances are determined for use by fuzzy interpolation.

Comparison of the results from those generated by the previous fuzzy hydrocyclone control model, and the same fuzzy model with the improved multidimensional fuzzy interpolation technique are shown in Table 1. In order to show the applicability of this proposed fuzzy hydrocyclone model, the results are also used to compare with results generated from the on-line control model as shown in [2]. The graphical plots of the results generated from the model with the improved multidimensional fuzzy interpolation technique are shown in Figure 4.

A few measurements of differences between the predicted d50c ( $T$ ) and observed d50c ( $O$ ) are used. They are: Euclidean Distance  $ED = \sqrt{\sum_{i=1}^P (T_i - O_i)^2}$ ; Mean

Character Difference Distance  $MCD = \frac{\sum_{i=1}^P |T_i - O_i|}{P}$ ; Percent Similarity Coefficient

$$PSC = 200 \frac{\sum_{i=1}^P \min(T_i, O_i)}{\sum_{i=1}^P (T_i + O_i)}$$

Table 1: Comparisons of results

Model Type	ED	MCD	PSC
Formula from [2]	59.787	5.179	90.096
Fuzzy (no fuzzy interpolation)	101.640	6.409	88.211
Fuzzy (with fuzzy interpolation)	52.97	4.595	91.889

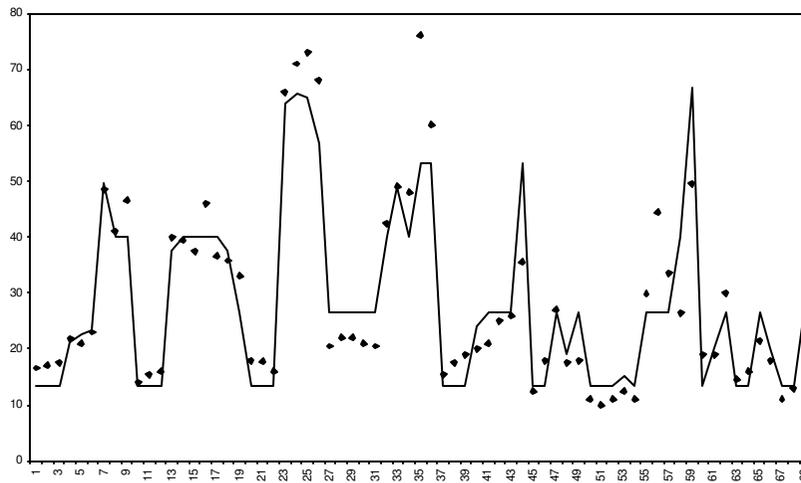


Figure 5: Output plot showing test case results with fuzzy interpolation.

From Table 1, the results show that the fuzzy hydrocyclone model performs unreasonably when no interpolation technique is used. This is mainly due to the four input instances that find no rule to fire and generate a default value of zero. With the fuzzy rule interpolation technique, the number of fuzzy rules is not increased, but the prediction ability has improved. This is a desirable characteristic for on-line hydrocyclone control, as an increase in number of fuzzy rules would result in an increase in complexity which would make the examination of the fuzzy rule base more difficult.

## 6. Conclusion

In this paper, the practical applicability of the self-generating fuzzy rule inference system in hydrocyclone control has been examined. The problem of sparse rule bases and insufficient input data may cause undesirable control actions. This is mainly due to input instances that could not find any rule in the fuzzy rule base. To provide a solution to this problem, the improved multidimensional fuzzy rule interpolation method has been applied. This method can be used to interpolate the gaps between the rules. This ensures that the set of sparse fuzzy rules generated by the self-generating fuzzy rule inference system will be useable in a practical system.

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